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Theoretical constraints on the vacuum oscillation solution to the solar neutrino problem

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Abstract: The vacuum oscillation (VO) solution to the solar anomaly requires an extremely small neutrino mass splitting, $\Delta m^2_{sol} \lesssim 10^{-10}$ eV^2. We study under which circumstances this small splitting (whatever its origin) is or is not spoiled by radiative corrections. The results depend dramatically on the type of neutrino spectrum. If $m_1^2 \sim m_2^2 \gtrsim m_3^2$, radiative corrections always induce too large mass splittings. Moreover, if m_1 and m_2 have equal signs, the solar mixing angle is driven by the renormalization group evolution to very small values, incompatible with the VO scenario (however, the results could be consistent with the small-angle MSW scenario). If m_1 and m_2 have opposite signs, the results are analogous, except for some small (though interesting) windows in which the VO solution may be natural with moderate fine-tuning. Finally, for a hierarchical spectrum of neutrinos, $m_1^2 \ll m_2^2 \ll m_3^2$, radiative corrections are not dangerous, and therefore this scenario is the only plausible one for the VO solution.

Keywords: Beyond Standard Model, Neutrino Physics.

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Contents

1. Introduction

There are three main explanations of the solar neutrino flux deficits, requiring oscillations of electron neutrinos into other species. Namely, the small and large angle MSW solutions, and the vacuum oscillation (VO) solution. In this paper we focus on the latter, which requires the relevant mass splitting and mixing angle in the range [1]

$$\begin{gathered} 5 \times 10^{-11}\ \mathrm{eV}^2 < \Delta m^2_{sol} < 1.1 \times 10^{-10}\ \mathrm{eV}^2\,, \\ \sin^2 2\theta_{sol} > 0.67\,. \end{gathered} \tag{1.1}$$

On the other hand, Super-Kamiokande observations [2] of atmospheric neutrinos require neutrino oscillations (more precisely $\nu_\mu - \nu_\tau$ oscillations if we do not consider oscillations into sterile species) driven by a mass splitting and a mixing angle in the range [1]

$$\begin{gathered} 5 \times 10^{-4}\ \mathrm{eV}^2 < \Delta m^2_{atm} < 10^{-2}\ \mathrm{eV}^2\,, \\ \sin^2 2\theta_{atm} > 0.82\,. \end{gathered} \tag{1.2}$$

Let us remark the enormous hierarchy of mass splittings[1] between the different species of neutrinos, $\Delta m^2_{sol} \ll \Delta m^2_{atm}$, which is apparent from eqs.(1.1) and (1.2).

It has been argued that the extreme tinyness of Δm^2_{sol} in this scenario could be related to some continuous or discrete symmetry at high energy [4]. However, independently of the origin of the small splittings, it must be required that their size is not spoiled by radiative corrections, the dominant part of which can be accounted by integrating the renormalization group equations (RGEs) between the scale at which the effective mass matrix is generated and low energy. The aim of this paper is to analyze under which circumstances this is in fact the case. As a result, we obtain important theoretical restrictions on the VO scenario.

[1] It has been pointed out [3] that disregarding Cl data on solar neutrinos, vacuum oscillations with much larger mass splitting could account for the solar anomaly.

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Let us introduce now some notation. We define the effective mass term for the three light (left-handed) neutrinos in the flavour basis, $\nu^T = (\nu_e, \nu_\mu, \nu_\tau)$, as

$$\mathcal{L} = -\frac{1}{2}\nu^T \mathcal{M}_\nu \nu \ + \ \text{h.c.} \tag{1.3}$$

The mass matrix, $\mathcal{M}_\nu$, is diagonalized in the usual way, i.e. $\mathcal{M}_\nu = V^* D V^\dagger$, where $D = \text{diag}(m_1 e^{i\phi}, m_2 e^{i\phi'}, m_3)$ and V is a unitary 'CKM' matrix, relating flavour to mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_2 c_3 & c_2 s_3 & s_2 e^{-i\delta} \\ -c_1 s_3 - s_1 s_2 c_3 e^{i\delta} & c_1 c_3 - s_1 s_2 s_3 e^{i\delta} & s_1 c_2 \\ s_1 s_3 - c_1 s_2 c_3 e^{i\delta} & -s_1 c_3 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{1.4}$$

Here, s_i and c_i denote $\sin\theta_i$ and $\cos\theta_i$, respectively, and in the rest of the paper we will neglect CP-violating phases. In the following, we label the mass eigenstates ν_i in such a way that $|\Delta m^2_{12}| < |\Delta m^2_{23}| \sim |\Delta m^2_{13}|$, where $\Delta m^2_{ij} \equiv m^2_j - m^2_i$ ($m^2_{\nu_3}$ is thus the most split eigenvalue). Consequently, Δm^2_{sol}, θ_{sol} correspond to Δm^2_{12}, θ_3, while Δm^2_{atm}, θ_{atm} correspond to $\Delta m^2_{23} \sim \Delta m^2_{13}$, θ_1 respectively. In our notation Δm^2_{sol}, Δm^2_{atm} denote always the "experimental" splittings of eqs.(1.1) and (1.2), while Δm^2_{12}, Δm^2_{23} denote the computed splittings once the radiative corrections are incorporated. Concerning the θ_2 angle, according to the most recent combined analysis of SK + CHOOZ data (last paper of ref. [1]) it is constrained to have low values, $\sin^2 2\theta_2 < 0.36\,(0.64)$ at 90% (99%) C.L.

We assume along the paper that the effective mass matrix for the left-handed neutrinos, $\mathcal{M}_\nu$, is generated at some high energy scale, Λ, by some unspecified mechanism. Below Λ, we consider two possibilities: either the effective theory is the Standard Model (SM) or the minimal supersymmetric Standard Model (MSSM) with unbroken $R-$parity. In the first case, the lowest dimension operator producing a mass term of this kind is uniquely given by [5]

$$-\frac{1}{4}\kappa \nu^T \nu H H + \text{h.c.} \tag{1.5}$$

where κ is a matricial coupling and H is the ordinary (neutral) Higgs. Obviously, $\mathcal{M}_\nu = \frac{1}{2}\kappa\langle H\rangle^2$. The effective coupling κ runs with the scale from Λ to M_Z, with a RGE given by [6]

$$16\pi^2 \frac{d\kappa}{dt} = \left[-3g_2^2 + 2\lambda + 6Y_t^2 + 2\text{Tr}\mathbf{Y_e^\dagger Y_e}\right]\kappa - \frac{1}{2}\left[\kappa \mathbf{Y_e^\dagger Y_e} + (\mathbf{Y_e^\dagger Y_e})^T \kappa\right], \tag{1.6}$$

where $t = \log\mu$, and $g_2, \lambda, Y_t, \mathbf{Y_e}$ are the SU(2) gauge coupling, the quartic Higgs coupling, the top Yukawa coupling and the matrix of Yukawa couplings for the charged leptons, respectively. The last term of eq. (1.6) is the most important one for our purposes, since it modifies the texture of $\mathcal{M}_\nu$. It is worth noticing that the modification

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of a mass eigenvalue is always proportional to the mass eigenvalue itself. In the MSSM case, things are very similar but with an important difference. Namely, the term that modifies the texture in the RGEs has the same form as in eq. (1.6) but with coefficient $+1$ instead of $-1/2$. Moreover, in the MSSM the $\mathbf{Y_e}$ couplings are $1/\cos\beta$ larger than the SM ones. All this implies that the effect of the RGEs in the supersymmetric case is $2/\cos^2\beta = 2(1+\tan^2\beta)$ times larger (for $\tan\beta = 2$ this already represents one order of magnitude). It should be mentioned that in the supersymmetric case there are two stages of running: from Λ to $M_{\rm SUSY}$ with the MSSM RGEs, and from $M_{\rm SUSY}$ to M_Z with the SM ones (the latter is normally much less important than the former).

In order to study the quantitative effect of the RGEs on the mass splittings and mixing angles, it is convenient to consider separately the following three possible scenarios [7]

$$\begin{aligned} &\mathrm{A}: |m_3| \gg |m_{1,2}|\,, \\ &\mathrm{B}: |m_1| \sim |m_2| \sim |m_3|\,, \\ &\mathrm{C}: |m_1| \sim |m_2| \gg |m_3|\,. \end{aligned} \tag{1.7}$$

In case A, radiative corrections are generically not dangerous. The reason is that, as stated before, the mass eigenvalues renormalize proportionally to themselves, i.e. $\Delta_{RGE}\, m_i = (K_0 + K_i) m_i$, where K_0 is the universal contribution for all the eigenvalues and $|K_i| \ll 1$. Thus, unless $m_{1,2}^2 \gg \Delta m_{12}^2$, the running cannot spoil the initial smallness of the solar mass-splitting (this is in particular the case of a hierarchical spectrum $m_1^2 \ll m_2^2 \ll m_3^2$). Roughly speaking, the mass splittings generated radiatively get larger than the allowed range of eq. (1.1) for $m_{1,2}^2 \sim 10^{-4}$ eV^2, although the precise value depends on several details, in particular on the values of the mixing angles.

On the other hand, case B (cosmologically relevant for $m_i = \mathcal{O}(\mathrm{eV})$), has been shown to be problematic for the VO solution in refs. [8, 9, 10]. Namely the mass splittings Δm_{ij}^2 generated through the running are generically[2] several orders of magnitude larger than the required VO splitting, even for Λ very close to M_Z. According to the previous discussion, the supersymmetric case works even worse.

Finally, the impact of the radiative corrections on a spectrum of the type C has not been considered yet in the literature. Since in this case the large $\Delta m_{sol}^2 \ll \Delta m_{atm}^2$ hierarchy forces $m_{1,2}^2 \gg \Delta m_{12}^2$, one can expect important radiative effects. The analysis of this scenario is performed in section 2, where we study in two separate subsections the possibilities that m_1 and m_2 have equal or opposite signs. The conclusions are presented in section 3.

[2]There is an interesting instance in which this is not the case. See note added at the end of the paper.

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2. The case $m_1^2 \sim m_2^2 \gg m_3^2$

As explained in the introduction, radiative corrections play an important rôle when $m_1^2 \sim m_2^2 \gg m_3^2$, case in which the mass splitting relevant for solar oscillations is the one between the heavier neutrinos. In this framework, radiative corrections can actually make $\Delta m_{12}^2 \gg \Delta m_{sol}^2$ in contradiction with observations. This effect will be stronger the heavier is the overall neutrino mass scale: the most conservative case thus corresponds to $m_1^2 \sim m_2^2 \sim \Delta m_{atm}^2$ and $m_3^2 \sim 0$ (masses smaller than this cannot accommodate atmospheric oscillations of the required frequency).

The rationale is then the following: at some high-energy scale Λ one assumes that new physics generates a dimension-5 operator leading to non-zero neutrino masses and fixes $\mathcal{M}_\nu(\Lambda)$ such that, with good approximation $m_1^2 = m_2^2$ and $m_3^2 = 0$. The most important radiative corrections to this tree-level masses are proportional to $\ln(\Lambda/M_Z)$ and can be included using standard RG techniques, that is, running $\mathcal{M}_\nu$ down from Λ to M_Z using the relevant RGEs. The latter depends on what is the effective theory below Λ. As we said, we consider two cases: SM and MSSM, and the RGEs relevant for these two effective theories can be found e.g. in ref. [6].

The analytical integration of the RGEs is straightforward in the leading-log approximation and the additional simplification that $m_3 \sim 0$ at all scales permits us to concentrate on the two other masses alone. The results are qualitatively different depending on the relative sign between m_1 and m_2 and we consider the two cases separately in the next subsections.

2.1 The case $m_1 \simeq m_2$

After integration from Λ to M_Z, the radiatively corrected $\mathcal{M}_\nu(M_Z)$ has eigenvalues which in first approximation are given by

$$\begin{aligned} m_1 &= m_\nu\,, \\ m_2 &= m_\nu\left[1 + 2\epsilon_\tau(1 - c_1^2 c_2^2)\right], \\ m_3 &= 0\,. \end{aligned} \tag{2.1}$$

These expressions include the leading-log radiative corrections to the mass differences and are obtained under the approximation that the initial 1-2 mass splitting is zero. In eq. (2.1), the family-universal renormalization effect (not important for our discussion) has been absorbed in m_ν, which is fixed to give the proper value for $\Delta m_{31}^2 \sim \Delta m_{atm}^2$. The θ_1, θ_2, θ_3 angles have been kept as free parameters. Our numerical results are always obtained integrating numerically the RGEs and confirm that the analytical expressions we write represent an excellent approximation. For our numerics we choose both the lower and upper limits of the allowed range for Δm_{atm}^2, thus fixing $m_\nu = 2.2 \times 10^{-2}$ eV or 0.1 eV (corresponding to $\Delta m_{31}^2 = 5 \times 10^{-4}\,\text{eV}^2$ and $10^{-2}\,\text{eV}^2$, respectively).

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The parameter ϵ_τ depends on what is the effective low-energy theory below Λ [9, 10]:

$$\epsilon_\tau = \frac{h_\tau^2}{32\pi^2}\log\frac{\Lambda}{M_Z} \qquad (\mathrm{SM})\,, \tag{2.2}$$
$$\epsilon_\tau = \frac{h_\tau^2}{32\pi^2}\left[-\frac{2}{\cos^2\beta}\log\frac{\Lambda}{M_{SUSY}}+\log\frac{M_{SUSY}}{M_Z}\right] \qquad (\mathrm{MSSM})\,, \tag{2.3}$$

where M_{SUSY} sets the mass scale for the supersymmetric spectrum (we take $M_{SUSY} \sim 1$ TeV). As usual, the size of ϵ_τ grows logarithmically with the scale of new physics Λ (a conservative estimate we often make is to choose a low value $\Lambda = 1$ TeV). Also, for sufficiently large Λ/M_Z, the size of ϵ_τ is enhanced by a factor $2/\cos^2\beta = 2(1+\tan^2\beta)$ in the MSSM with respect to the SM (already a factor 10 for $\tan\beta = 2$) so that radiative corrections are more important in this case.

The typical size of ϵ_τ is $\sim 8\times 10^{-7}$ ($\sim 8\times 10^{-6}$) for $\Lambda = 10^3$ GeV ($\Lambda = 10^{12}$ GeV) in the SM and $\sim -8\times 10^{-5}$ for $\Lambda = 10^{12}$ GeV in the MSSM with $\tan\beta = 2$. According to eq. (2.1) this would lead to

$$\frac{\Delta m^2_{sol}}{m^2_\nu} = \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = 4\epsilon_\tau(1-c_1^2c_2^2)\,, \tag{2.4}$$

too large compared with the observed value unless there is a cancellation in $(1-c_1^2c_2^2)$, which requires $\sin^2 2\theta_{1,2} \sim 0$. This is far from the best-fit values mentioned in the introduction. Choosing $\sin^2 2\theta_1 \simeq 1$ and $\sin^2 2\theta_2 \simeq 0$ we must conclude that Δm^2_{12} turns out to be too large for vacuum oscillations of solar neutrinos.

The precise results are given in figure 1, which shows the predicted Δm^2_{12} in eV^2 (solid lines) as a function of $c_1^2c_2^2$ for the SM case with $\Lambda = 10^3$ GeV and $\Delta m^2_{atm} = 5\times 10^{-4}\,\mathrm{eV}^2$ (lower curve) and $10^{-2}\,\mathrm{eV}^2$ (upper curve). The experimental constraints on $\theta_{1,2}$ leave open the windows $0 \leq c_1^2c_2^2 \leq 0.142$ and $0.232 \leq c_1^2c_2^2 \leq 0.71$, as indicated. The neutrino mass splitting required by VO solar oscillations is marked by the horizontal dotted lines. As was clear from the previous discussion, there is no overlapping between the Δm^2_{12} predicted and the Δm^2_{sol} required. Indeed, Δm^2_{12} is always much larger than the allowed range. In the MSSM (or for larger Λ) the situation is even worse because in both cases Δm^2_{12} increases significantly in the way discussed above.

Let us turn in more detail to the mixing angles in this scenario. At the scale Λ one has some mixing angles θ_i which will be different in general at the scale M_Z after radiative corrections to $\mathcal{M}_\nu$ have been included. At the same level of approximation as in eqs. (2.1), the eigenvectors of the perturbed neutrino mass matrix are of the form

$$V_1' = \frac{1}{\sqrt{\alpha^2+\beta^2}}(\alpha V_1+\beta V_2)\,, \qquad V_2' = \frac{1}{\sqrt{\alpha^2+\beta^2}}(-\beta V_1+\alpha V_2)\,, \qquad V_3' = V_3\,, \tag{2.5}$$

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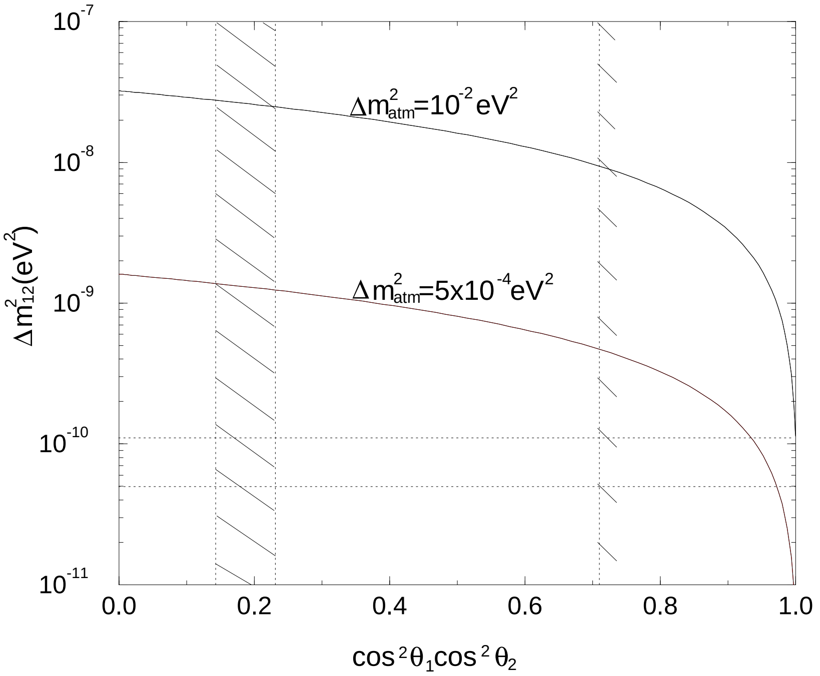

Figure 1: $\Delta m^2_{12}/\mathrm{eV}^2$ (solid lines) as a function of $c_1^2c_2^2$ for $\Delta m^2_{atm} = 10^{-2}\,\mathrm{eV}^2$ (upper curve) and $5\times 10^{-4}\,\mathrm{eV}^2$ (lower). The experimentally allowed region for $c_1^2c_2^2$ is delimited by the dashed lines (the dashed regions are forbidden) and the Δm^2_{sol} needed for the VO solution is given by the range between the dotted lines. This plot corresponds to the SM with $\Lambda = 10^3$ GeV.

where V_i are the eigenvectors corresponding to $\mathcal{M}_\nu(\Lambda)$

$$V_1 = \begin{pmatrix} c_2c_3 \\ -c_1s_3 - s_1s_2c_3 \\ s_1s_3 - c_1s_2c_3 \end{pmatrix}, \qquad V_2 = \begin{pmatrix} c_2s_3 \\ c_1c_3 - s_1s_2s_3 \\ -s_1c_3 - c_1s_2s_3 \end{pmatrix}, \qquad V_3 = \begin{pmatrix} s_2 \\ s_1c_2 \\ c_1c_2 \end{pmatrix}. \tag{2.6}$$

From this, we deduce that the relationships between $\theta_i(M_Z)$ and $\theta_i(\Lambda)$ are

$$\begin{aligned} \sin^2 2\theta_1(M_Z) &= \sin^2 2\theta_1(\Lambda), \\ \sin^2 2\theta_2(M_Z) &= \sin^2 2\theta_2(\Lambda), \\ \sin^2 2\theta_3(M_Z) &= \sin^2 2\theta_3(\Lambda) + \frac{2}{(1+r)^2}\left[\sqrt{r}(1-r)\sin 4\theta_3(\Lambda) + 2r\cos 4\theta_3(\Lambda)\right], \end{aligned} \tag{2.7}$$

where $r \equiv \alpha^2/\beta^2$. In leading-log approximation we have

$$\frac{\alpha}{\beta} \simeq \frac{s_1c_3 + c_1s_2s_3}{s_1s_3 - c_1s_2c_3}, \tag{2.8}$$

with all angles evaluated at the scale Λ.

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If we substitute this in (2.7) we find the simpler expression

$$\sin^2 2\theta_3(M_Z) = \frac{\sin^2\theta_2 \sin^2 2\theta_1}{(1-\cos^2\theta_1\cos^2\theta_2)^2}\,. \tag{2.9}$$

For the bimaximal mixing case ($s_2 \sim 0$, $c_1 \sim s_1 \sim 1/\sqrt{2}$) we end up with $\sin^2 2\theta_3(M_Z) \sim 0$, which is not acceptable (observations require $\sin^2 2\theta_3 \geq 0.67$).

In conclusion, the scenario $m_1 \sim m_2 \gg m_3$ is very contrived from the theoretical point of view. It is not natural to expect in this framework the values of mass splittings and mixing angles which are suggested by experiment. As mentioned in the introduction, the only way-out would be an extremely artificial fine-tuning between the initial values of the mass splittings (and mixing angles) and the effect of the RG running. If one insists on this possibility, starting for example with $\Delta m^2_{atm} = 5\times 10^{-4}$ eV^2, $s_2 \sim 0$, $c_1 \sim s_1 \sim 1/\sqrt{2}$, $\Lambda = 10^3$ GeV (a conservative choice for the fine-tuning problem), one is forced to take the initial mass splitting and mixing angle within the narrow ranges $|m_1 - m_2| \sim (1.82\pm 0.02)\times 10^{-7}$ eV and $\theta_3 \sim \pi/2 \pm 5.5\times 10^{-3}$ in order to compensate the effect of the RGEs and reproduce the required pattern of masses and mixings at M_Z (these numbers cannot be extracted from the previous eq. (2.4), as in this case the approximation of initial degenerate eigenvalues does not hold). One cannot certainly expect such a conspiracy between totally unrelated effects. If one slightly separates from these narrow ranges the low-energy mass splitting would be much larger than the required one. Of course, as Δm^2_{atm} or Λ are raised, or one goes to the supersymmetric case, the fine-tuning becomes much stronger.

Finally, it is interesting to note that for sizeable values of the cut-off ($\Lambda \gtrsim 10^{12}$ GeV) and/or a supersymmetric scenario, the values of Δm^2_{12} are naturally 1-3 orders of magnitude larger than those represented in Fig.1, falling in the small-angle MSW range (3×10^{-6} eV^2 $< \Delta m^2_{sol} < 10^{-5}$ eV^2). This is appealing since, as has been discussed in this section, starting with $\theta_{1,2}$ mixing angles in agreement with experiment ($s_2 \sim 0$, $c_1 \sim s_1 \sim 1/\sqrt{2}$) the RGEs drive $\sin^2 2\theta_3(M_Z) \sim 0$, independently of its initial high-energy value, see eq. (2.7). This is exactly what is needed for a successful small-angle MSW solution to the solar neutrino problem.

2.2 The case $m_1 \simeq -m_2$

In this case, the neutrino mass eigenvalues at M_Z are, in leading-log approximation

$$\begin{aligned} m_1 &= m_\nu \left[1 + 2\epsilon_\tau (s_1 s_3 - c_1 s_2 c_3)^2\right], \\ m_2 &= -m_\nu \left[1 + 2\epsilon_\tau (s_1 c_3 + c_1 s_2 s_3)^2\right], \\ m_3 &= 0\,, \end{aligned} \tag{2.10}$$

with ϵ_τ as given by eqs. (2.2) and (2.3). The mixing angles are, in first approximation, equal at M_Z and Λ. We fix again $m_\nu \sim \sqrt{\Delta m^2_{atm}}$. In order not to spoil the size of

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the required solar mass splitting, the radiative corrections should generate $\Delta m^2_{12} \leq \Delta m^2_{sol}$. The prediction from (2.10) is

$$\frac{\Delta m^2_{12}}{m^2_\nu} = \frac{\Delta m^2_{12}}{\Delta m^2_{atm}} = 4\epsilon_\tau \left[(\cos^2\theta_1 \sin^2\theta_2 - \sin^2\theta_1)\cos 2\theta_3 - \sin 2\theta_1 \sin\theta_2 \sin 2\theta_3 \right] . \tag{2.11}$$

Getting a sufficiently small number for this quantity requires some (in general delicate) correlation between the mixing angles, in such a way that

$$\tan 2\theta_3 \simeq \frac{\cos^2\theta_1 \sin^2\theta_2 - \sin^2\theta_1}{\sin\theta_2 \sin 2\theta_1} . \tag{2.12}$$

It is remarkable that the bimaximal values of the mixing angles ($\sin^2 2\theta_1 \sim \sin^2 2\theta_3 \sim 1$ and $\sin^2 2\theta_2 \sim 0$) do satisfy (2.12).

Figures 2 show, for the SM case, the regions in the plane $(\sin^2 2\theta_2, \sin^2 2\theta_3)$ where the correlation (2.12) takes place, giving $\Delta m^2_{12} \leq 1.1 \times 10^{-10}\,\mathrm{eV}^2$ (the upper limit on Δm^2_{sol}). The width of these allowed regions is controlled by $\Delta m^2_{sol}/(\epsilon_\tau \Delta m^2_{atm})$. The larger $\epsilon_\tau \Delta m^2_{atm}$ (or the smaller Δm^2_{sol}), the thinner these regions get (because a more delicate cancellation must take place in (2.12)). In figure 2*a* we have fixed $\sin^2 2\theta_1 = 1$ and $\Delta m^2_{atm} = 5 \times 10^{-4}\,\mathrm{eV}^2$, and we give the allowed areas for the two choices $\Lambda = 10^3$ GeV (thick region, delimited by the outermost lines) and 10^{12} GeV (thin region). If we choose $\Delta m^2_{atm} = 10^{-2}\,\mathrm{eV}^2$ instead, the two regions would shrink significantly and will be somewhere inside the thin region shown for $\Lambda = 10^{12}$ GeV. The dashed lines delimit the allowed region for the two mixing angles θ_2 and θ_3 ($0 \leq \sin^2 2\theta_2 \leq 0.64$ and $0.67 \leq \sin^2 2\theta_3 \leq 1$). Figure 2*b* corresponds to the case $\sin^2 2\theta_1 = 0.82$ (the lower experimental limit) and the same values of other parameters as in figure 2*a*. The results are similar except for a shift towards smaller $\sin^2 2\theta_2$ values in the region of interest. Note in particular that the upper limit $\sin^2 2\theta_2 \sim 0.64$ is never reached in this scenario. We see that in the most conservative case, with $\Delta m^2_{atm} = 5 \times 10^{-4}\,\mathrm{eV}^2$ and $\Lambda = 10^3$ GeV, a significant portion of parameter space could accommodate a Δm^2_{12} of the right order of magnitude (including the bimaximal mixing solution). It is interesting to note that inside this region, starting with degenerate m_1, m_2 would lead to a correct Δm^2_{sol} at low energy, thus providing a dynamical origin for this small number. Notice however that as soon as Δm^2_{atm} or Λ are raised the required fine-tuning becomes much stronger. This occurs in particular if the lower bound $\Delta m^2_{atm} = 5 \times 10^{-4}$ that we have used is increased according to the analyses of the most recent data [11].

The situation is worse in the MSSM case. Roughly speaking, for $\tan\beta = 3$ radiative corrections are 20 times larger than in the SM (with the same Λ). The cancellation between mixing angles in (2.12) is thus much more delicate in the supersymmetric case, as expected.

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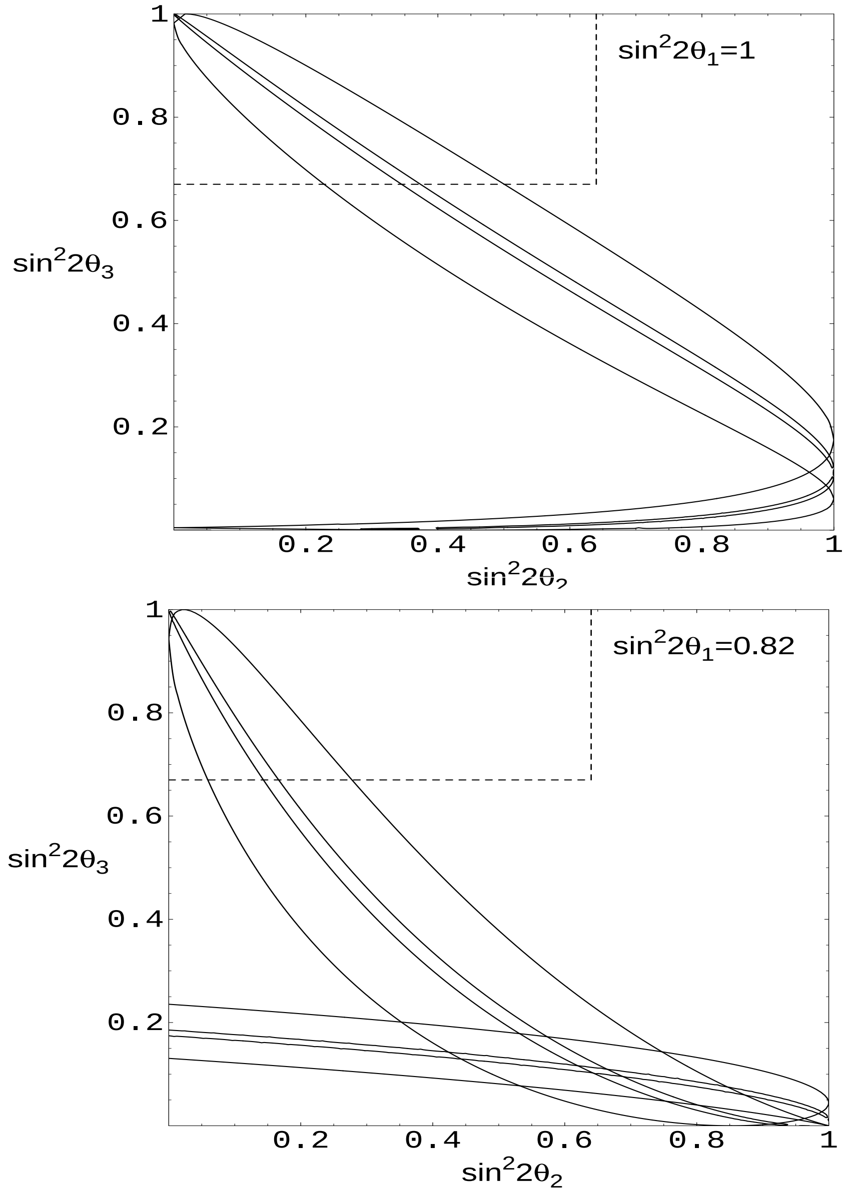

Figure 2: Allowed regions (in which $\Delta m^2_{12} \leq 1.1 \times 10^{-10}\,\mathrm{eV}^2$) in the plane $(\sin^2 2\theta_2, \sin^2 2\theta_3)$ for the SM case for $\sin^2 2\theta_1 = 1$ (upper plot) and 0.82 (lower). The area between the outermost (innermost) lines corresponds to a cut-off scale $\Lambda = 10^3$ GeV (10^{12} GeV). The dashed lines delimit the experimentally allowed values for $\sin^2 2\theta_2$ and $\sin^2 2\theta_3$.

3. Conclusions

The vacuum oscillation (VO) solution to the solar neutrino problem requires an extremely small mass splitting, $\Delta m^2_{sol} \lesssim 10^{-10}$ eV^2. We have studied in this paper under which circumstances this smallness (whatever its origin) is or is not spoiled by radiative corrections, in particular by the running of the renormalization group equations (RGEs) between the scale at which the effective neutrino mass matrix

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is generated (Λ) and low energy. We consider the cases where the effective theory below Λ is the Standard Model (SM) or the Minimal Supersymmetric Standard Model (MSSM). The results depend dramatically on the type of neutrino spectrum. In particular, if $m_1^2 \ll m_2^2 \ll m_3^2$, radiative corrections are always relatively small and do not cause any significant change in the splittings. On the other hand, if $m_1^2 \sim m_2^2 \sim m_3^2$, radiative corrections generically[3] induce mass splittings that are several orders of magnitude larger than the required Δm^2_{sol}. Hence, in our opinion this type of spectrum is not plausible for the VO solution.

Most of the paper is devoted to the third possible type of spectrum, $m_1^2 \sim m_2^2 \gg m_3^2$, which requires $m_{1,2}^2 \sim \Delta m^2_{atm}$ (or larger). Here again, the radiatively generated splittings are in general too large, making the scenario unnatural. As a general rule, this gets worse as Δm^2_{atm} or Λ grow. Also, the supersymmetric scenario works worse than the SM one, especially as $\tan\beta$ incresases. More precisely, if m_1 and m_2 have equal signs, the RGE-induced splittings are always too large, even for the most favorable case. In addition, the solar mixing angle is driven by the RGEs to very small values, $\sin^2 2\theta_3(M_Z) \sim 0$, which is incompatible with the VO solution. It is however worth noticing that such a small angle is what is needed for a successful small-angle MSW solution to the solar neutrino problem. Moreover, for $\Lambda \gtrsim 10^{12}$ GeV and/or for the MSSM scenario the values of Δm^2_{12} may fall naturally in the small-angle MSW range, $\sim 10^{-5}$ eV^2.

If m_1 and m_2 have opposite signs, the results are analogous, but now the splitting generated by the RGEs can vanish if the mixing angles are correlated in a particular way (which remarkably is always satisfied by the exact bimaximal case). This correlation or tuning of parameters is acceptable in the SM scenario, provided the cut-off scale Λ is not much larger than ~ 1 TeV and if Δm^2_{atm} is in the low side of its experimentally preferred range ($\sim 5\times 10^{-4}\,\mathrm{eV}^2$). Interestingly, this could provide a dynamical origin for the smallness of Δm^2_{sol}. For larger Λ and/or Δm^2_{atm} (or equivalently, for the MSSM scenario) radiative corrections grow in size and the required tuning of mixing angles becomes quickly unacceptable. This occurs in particular if the lower bound $\Delta m^2_{atm} = 5\times 10^{-4}$ that we have used is increased according to the most recent data analyses [11].

In conclusion, apart from the mentioned small windows, a completely hierarchical spectrum of neutrinos (i.e. as the spectrum of quarks and charged leptons), $m_1^2 \ll m_2^2 \ll m_3^2$, seems to be the only plausible one for the VO solution to the solar neutrino problem.

Note added. After the first version of this paper appeared, there has come out a paper by Barbieri, Ross and Strumia (`hep-ph/9906470`) on a similar subject. Their results for the $m_1^2 \sim m_2^2 \gg m_3^2$ case, which is the central topic of the present paper, fully agree with ours.

[3] See however note added below.

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For the $m_1^2 \sim m_2^2 \sim m_3^2$ case (studied in [8, 9, 10] and mentioned in this paper), they find a scenario in which the radiative corrections to the VO mass splittings are strongly suppressed. Namely, this happens when the initial condition is $\Delta m_{32}^2 \sim \Delta m_{atm}^2 \gg \Delta m_{12}^2$. Moreover, the mixing angles must fulfill $\theta_2 = 0$, $\theta_3 = \pi/4$ with extreme accuracy. This represents a tiny region of the parameter space although it could be consequence of some symmetry, as commented by the authors.

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